Give the **complete definition** of the definite integral as presented in lecture. **SCORE: ______ / 4 PTS**

SEE HANDOUT ON MY WEBSITE

Use the **definition** of the definite integral and the theorem about continuous integrands to evaluate $\int_{-1}^{3} (2x-4)\,dx$. **SCORE: ______ / 9 PTS**

NOTE: 0 points if you use the Fundamental Theorem of Calculus or geometry instead.

REMEMBER THIS OVERRIDING RULE

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(-1+\frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left[2\left(-1+\frac{4i}{n}\right)-4\right]$$

$$= \lim_{n\to\infty} \frac{4}{n} \sum_{i=1}^{n} \left(-6+\frac{8i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{4}{n} \left[\sum_{i=1}^{n} -6 + \frac{8}{n}\sum_{i=1}^{n} i\right]$$

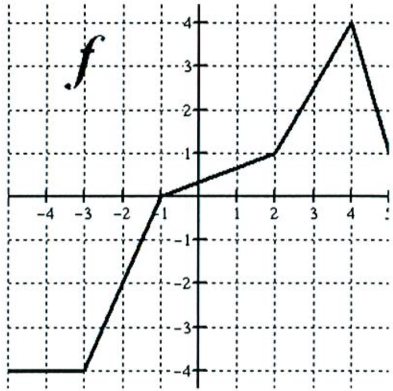
$$= \lim_{n\to\infty} \frac{4}{n} \left[-6n + \frac{8}{n}\,\frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} 4\left[-6+\frac{4(n+1)}{n}\right]$$

$$= 4(-6+4)$$

$$= -8$$

The graph of function f is shown on the right. **SCORE: ______ / 7 PTS**



[a] Find $\int_{-3}^{4} f(t)\,dt$.

$$= \int_{-3}^{-1} f(t)\,dt + \int_{-1}^{2} f(t)\,dt + \int_{2}^{4} f(t)\,dt$$

$$\underset{2}{=} \underbrace{-\tfrac{1}{2}(4)(2)} + \underbrace{\tfrac{1}{2}(1)(3)}_{1} + \underbrace{\left(\tfrac{1+4}{2}\right)2}_{1}$$

$$= -4 + \frac{3}{2} + 5$$

$$= \underbrace{\frac{5}{2}}\ 1$$

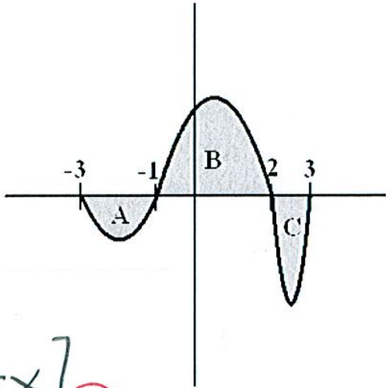
[b] Find $\int_{5}^{4} f(t)\,dt$.

$$= -\int_{4}^{5} f(t)\,dt$$

$$= -\left(\frac{4+1}{2}\right)1 \underset{1}{=} \boxed{-\frac{5}{2}}\ 1$$

The graph of $f(x)$ is shown on the right. If the area of shaded region A is 4, the area of shaded region B is 9, and the area of shaded region C is 3, find $\int_{-3}^{3} (5 - 7f(x))\,dx$.

SCORE: ______ / 6 PTS



<u>For full credit, you must clearly show the use of all necessary properties of the definite integral.</u>
<u>Minimal credit will be given for arithmetic alone.</u>

$$= \int_{-3}^{3} 5\,dx - \int_{-3}^{3} 7f(x)\,dx \quad 1$$

$$= 1\; 5(3 - {}^{-}3) - 7\int_{-3}^{3} f(x)\,dx \quad 1$$

$$= 30 - 7\left[\int_{-3}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx + \int_{2}^{3} f(x)\,dx\right] \quad 1$$

$$= 30 - 7[-4 + 9 - 3] \quad 1$$

$$= 16 \quad 1$$

Sketch a region whose area is given by the expression $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}e^{-2+\frac{3i}{n}}$. Label your graph clearly. **SCORE: ______ / 4 PTS**

$\Delta x = \frac{b-a}{n} = \frac{3}{n} \longrightarrow b-a=3$

$f(a+i\Delta x) = f(a+\frac{3i}{n}) = e^{-2+\frac{3i}{n}}$

$a=-2 \longrightarrow b=1$

$f(x)=e^x$